# ARE LONGITUDINAL FORCES PREDICTED IN MAGNETOSTATICS BY THE AMPÈRE FORCE LAW IN ITS LINE-CURRENT-ELEMENT FORM?

C. CHRISTODOULIDES
*Department of Physics, National Technical University of Athens, Zografou Campus, GR 157 73 Zografou, Athens, Greece*

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It is proved that a complete line-current loop exerts, according to the Ampère force law in magnetostatics, a force on a line-current element which is normal to it, irrespectively whether or not the element belongs to the current loop exerting the force. It is therefore shown that the so-called longitudinal Ampère forces do not exist.

There has been a controversy recently over the question of which of the two force laws in magnetostatics, Ampère's or that of Biot–Savart, is the correct one to use when evaluating forces of a current loop on parts of itself [1]. There have been claims that Ampère's law predicts forces exerted by the loop on a current element which are longitudinal, i.e. act along the current element [1–3] and may be detected experimentally [1,3–5], thus giving the correct theoretical interpretation, in contrast to the Biot–Savart law which fails in this respect and should therefore be discarded, with serious consequences on the validity of the special theory of relativity.

Proof that the two laws are equivalent in magnetostatics and preclude the existence of longitudinal forces [6–8] when used in their correct volume-current-element form and not the line-current-element form, as well as alternative theoretical interpretation of phenomena such as wire fragmentation by high currents [9,10] do not appear to have settled the matter [11–13].

Far from questioning the correctness of the proofs of equivalence of the two laws, it is our intention to present a simple proof of the fact that even in its line-current-element form, Ampère's law does not predict longitudinal forces. For this purpose we consider a closed space curve C (fig. 1) as representing the current loop. At the origin O there is a line-current element $I'$ d$\boldsymbol{l}$. The line current element $I$ d$\boldsymbol{r}$ at the position $\boldsymbol{r}$ exerts a force on d$\boldsymbol{l}$, which according

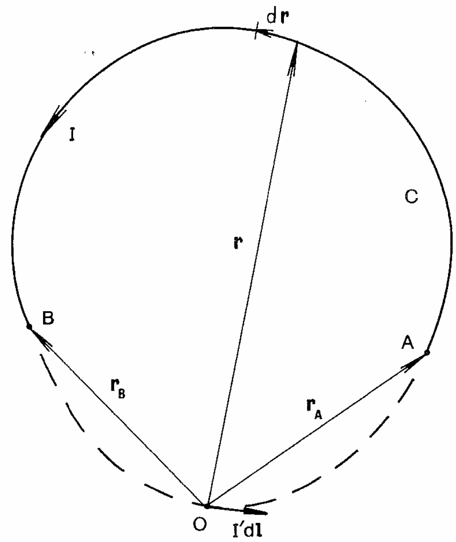


Fig. 1. The current loop C and the line current element $I'$ d$\boldsymbol{l}$.

to Ampère's force law is

$$d^2\boldsymbol{F}=\frac{\mu_0 I I'}{4\pi}\frac{\boldsymbol{r}}{r^3}\left(2(d\boldsymbol{l}\cdot d\boldsymbol{r})-\frac{3}{r^2}(d\boldsymbol{l}\cdot\boldsymbol{r})(\boldsymbol{r}\cdot d\boldsymbol{r})\right). \quad (1)$$

Taking the scalar product of this with d$\boldsymbol{l}$ we have

$$d\boldsymbol{l}\cdot d^2\boldsymbol{F}=\frac{\mu_0 I I'}{4\pi}\left(\frac{2}{r^3}(d\boldsymbol{l}\cdot\boldsymbol{r})(d\boldsymbol{l}\cdot d\boldsymbol{r})-\frac{3}{r^4}(d\boldsymbol{l}\cdot\boldsymbol{r})^2 dr\right), \quad (2)$$

$$\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}^2\boldsymbol{F}=\frac{\mu_0 II'}{4\pi}\,\mathrm{d}\left(\frac{(\mathrm{d}\boldsymbol{l}\cdot\boldsymbol{r})^2}{r^3}\right), \tag{3}$$

with d$\boldsymbol{l}$ being constant.

The Ampère force on d$\boldsymbol{l}$ exerted by the part of the loop from A to B (moving in the direction of $I$), has a scalar product with d$\boldsymbol{l}$ equal to

$$\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}\boldsymbol{F}(\mathrm{AB})=\int_{\mathrm{A}}^{\mathrm{B}}\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}^2\boldsymbol{F}$$

$$=\frac{\mu_0 II'}{4\pi}\left(\frac{(\mathrm{d}\boldsymbol{l}\cdot\boldsymbol{r}_{\mathrm{B}})^2}{r_{\mathrm{B}}^3}-\frac{(\mathrm{d}\boldsymbol{l}\cdot\boldsymbol{r}_{\mathrm{A}})^2}{r_{\mathrm{A}}^3}\right). \tag{4}$$

Since C is a closed loop there is no discontinuity of the integrand in eq. (4).

When d$\boldsymbol{l}$ does not lie on C, and if A and B coincide, eq. (4) reduces to

$$\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}\boldsymbol{F}(\mathrm{C})=\oint_{\mathrm{C}}\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}^2\boldsymbol{F}=0\,, \tag{5}$$

showing that a complete current loop C exerts an Ampère force d$\boldsymbol{F}$(C) on a current element not lying on the loop, which is normal to the element.

For d$\boldsymbol{l}$ on the loop, we take the origin at d$\boldsymbol{l}$ and express the curve C by the expansion

$$\boldsymbol{r}(s)=\boldsymbol{r}(\mathrm{O})+\boldsymbol{r}'(\mathrm{O})s+\tfrac{1}{2}\boldsymbol{r}''(\mathrm{O})s^2$$

$$+\tfrac{1}{6}\boldsymbol{r}'''(\mathrm{O})s^3+\ldots \tag{6}$$

in terms of the distance $s$ from the origin, measured along the curve and being positive in the direction of $I$. The derivatives of $\boldsymbol{r}$ are with respect to $s$ and evaluated at the origin. Then,

$$\boldsymbol{r}(\mathrm{O})=0\,, \tag{7}$$

$$\boldsymbol{r}'(\mathrm{O})=\boldsymbol{T}\,, \tag{8}$$

$$\boldsymbol{r}''(\mathrm{O})=\kappa\boldsymbol{N}\,, \tag{9}$$

$$\boldsymbol{r}'''(\mathrm{O})=-\kappa^2\boldsymbol{T}+\kappa'\boldsymbol{N}+\kappa\tau\boldsymbol{B}\,, \tag{10}$$

etc., with $\boldsymbol{T}$, $\boldsymbol{N}$, and $\boldsymbol{B}$ the unit tangent, principal normal and binormal vectors at O respectively and $\kappa$ the curvature, $\tau$ the torsion and $\kappa'=\mathrm{d}\kappa/\mathrm{d}s$ for the curve at O. Taking $\boldsymbol{r}_{\mathrm{A}}=\boldsymbol{r}(s_0)$ and $\boldsymbol{r}_{\mathrm{B}}=\boldsymbol{r}(-s_0)$, i.e. A and B at distance $s_0$ from O along the curve in opposite directions, expanding the terms of eq. (4) in series of ascending powers of $s_0$ keeping the denominators positive, and with $\mathrm{d}\boldsymbol{l}=\mathrm{d}l\,\boldsymbol{T}$ and $I'=I$, we obtain

$$\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}\boldsymbol{F}(\mathrm{AB})=\frac{\mu_0(I\,\mathrm{d}l)^2}{16\pi}\kappa\kappa' s_0^2+\ldots\,, \tag{11}$$

where higher powers of $s_0$ have been neglected. It is seen that as A and B approach O and coincide closing the loop with d$\boldsymbol{l}$ on it, i.e. $s_0\to\mathrm{O}$, then

$$\mathrm{d}\boldsymbol{l}\cdot\mathrm{d}\boldsymbol{F}(\mathrm{C})=0\,. \tag{12}$$

This shows that the Ampère force from a complete current loop on a line-current element is always normal to the latter, both when it does and when it does not belong to the current loop exerting the force. The longitudinal Ampère force must therefore be considered as a result of improper application of Ampère's force law. The use of the law in the form given in eq. (1) is likely to create problems, as mentioned above, not the least of which is the fact that the forces exerted by the loop on parts of itself are found to diverge at all points except those on rectilinear parts of the current loop [6,14]. For this particular purpose, use of the law in this form should therefore be avoided in favour of its volume-current-element formulation, which also givgidentical results with the Biot–Savart law in magnetostatics.

## References


[1] P. Graneau, J. Appl. Phys. 53 (1982) 6648.
[2] P. Graneau, Phys. Lett. A 97 (1983) 253.
[3] P. Graneau, J. Appl. Phys. 55 (1984) 2598.
[4] P.T. Pappas and P.G. Moyssides, Phys. Lett. A 111 (1985) 193.
[5] J. Nasilowski, Phys. Lett. A 111 (1985) 315.
[6] D.C. Jolly, Phys. Lett. A 107 (1985) 231.
[7] J.G. Ternan, J. Appl. Phys. 57 (1985) 1743.
[8] C. Christodoulides, J. Phys. A, to be published.
[9] H. Aspden, Phys. Lett. A 111 (1985) 22.
[10] J.G. Ternan, Phys. Lett. A 115 (1986) 230.
[11] P. Graneau, J. Appl. Phys. 58 (1985) 3638.
[12] J.G. Ternan, J. Appl. Phys. 58 (1985) 3639.
[13] P. Graneau and P.N. Graneau, Nuovo Cimento D 7 (1986) 31.
[14] C. Christodoulides, to be published.